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Exact time-localized solutions in Vacuum String Field Theory

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Abstract

We address the problem of finding star algebra projectors that exhibit localized time profiles. We use the double Wick rotation method, starting from a Euclidean (unconventional) lump solution, which is characterized by the Neumann matrix being the conventional one for the continuous spectrum, while the inverse of the conventional one for the discrete spectrum. This is still a solution of the projector equation and we show that, after inverse Wick-rotation, its time profile has the desired localized time dependence. We study it in detail in the low energy regime (field theory limit) and in the extreme high energy regime (tensionless limit) and show its similarities with the rolling tachyon solution.

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1 Introduction

The search for time-dependent solutions has lately become one of the prominent research topics in string theory. Particularly interesting is the search for solutions describing the decay of D-branes. An archetype problem in open bosonic string theory is describing the evolution from the maximum of the tachyon potential to the (local) minimum. Such a solution known as rolling tachyon, if it exists, describes the decay of the space filling D25-brane corresponding to the unstable perturbative vacuum to the locally stable vacuum. That such a solution exists has been argued in many ways, [1], see also [2, 3, 4]. A natural framework where to study such a nonperturbative problem is String Field Theory (SFT). But, although there have been some attempts to describe such phenomena in a SFT framework [5], no analytical control has been achieved so far.

In [22] we noticed that in this regard Vacuum String Field Theory (VSFT) could play an important role. Let us recall that VSFT, [6], is a version of Witten's open SFT, [7], that is supposed to describe the theory at the minimum of the tachyonic potential. There is evidence that at this point the negative tachyonic potential exactly compensates for the D25-brane tension. No open string mode is expected to be excited, so that the BRST cohomology must be trivial. This state can only correspond to the closed string vacuum.

Due to these properties VSFT is a simplified version of SFT. The BRST operator \mathcal{Q} takes a very simple form in terms of ghost oscillators alone. It is clearly simpler to work in such a framework than in the original SFT. In fact many classical solutions have been shown to exist, which are candidates for representing D-branes (the sliver, the butterfly, etc.), and other classical solutions have been found (lump solutions) which may represent lower dimensional D-branes [8, 9, 10, 11, 12, 13, 14].

In the present paper we show that the matter star algebra contains exact time-dependent projectors with the appropriate characteristic to represent S-branes, that is solitonic solutions localized in time. We show in fact that the time profile of such solutions is dominated for large t by a factor $\exp(-at^2)$ with positive constant a . At time $t = 0$ the solution takes the form of a deformed sliver (D25-brane), the deformation being parametrized by two continuous parameters. At infinite future (and infinite past) time it becomes 0, i.e. it flows into the stable vacuum. If the initial configuration happens to coincide exactly with the sliver (no deformation present) there cannot be any time evolution. Therefore an initial deformation away from the sliver is essential for true time evolution. Needless to say this is strongly reminiscent of Sen's rolling tachyon solution, [1] or of an S-brane, [16], i.e of a state finely tuned to be poised at the initial time near the top of the tachyon potential and let free to evolve.

The technique to produce such a solution is based on double Wick-rotation, as is customary in such kind of trade. Our reference solution is obtained by picking a Euclidean lump solution with one transverse space direction (a D24-brane) and then

performing an inverse Wick-rotation along such a direction. However the important ingredient is that our lump solution is not the ordinary one. Since the spectrum of the twisted Neumann coefficient matrices of the three strings vertex nicely split into a continuous and a discrete part, we define a new solution in which the squeezed state matrix is made of a continuous part, which is the same as for the conventional lump, and a discrete part which is *inverted* with respect to the ordinary lump. We call this *unconventional lump*, see eqs.(5.1,5.2) below. After inverse-Wick-rotating it we get the desired time behavior, (5.19).

In the previous paragraphs we have informally talked about time. Now we would like to be more precise. Our time is nothing but a Wick-rotated space coordinate, representing the position of the string center-of-mass, and it couples to the open string (flat) metric. In the conclusive section we will discuss a possible connection of such time with the closed string time (which couples to the bulk gravity metric). See [17, 18, 19, 20] for discussions related to the definition of time in SFT.

In this paper we limit ourselves to essentially one type of solution. It is obvious however that one can describe in the same way the decay of D_k-branes for any $k < 24$. There are also other types of solutions. They will be illustrated elsewhere together with complementary aspects of the present solution.

The paper is organized as follows. In section 2 we review a few basic VSFT formulas and briefly introduce the idea of *inverse sliver* and *inverse lump*. In section 3 we review the diagonal representation in VSFT and write down some useful approximation formulas. Section 4 is of pedagogical nature: we show that if we use the above recipe starting from an ordinary lump, we do not get anywhere. This illustrates the need to start from unconventional lumps. In section 5 we deal with the main problem, that is showing that starting from an unconventional lump we arrive at the desired time-dependent solution. In section 6 we describe the low energy and tensionless limits. Section 7 is devoted to a discussion of the results.

2 Sliver, inverse sliver and lumps

In order to render this paper as self-contained as possible, in this section we collect many well-known formulas and results concerning VSFT. The action is

$$\mathcal{S}(\Psi) = -\frac{1}{g_0^2} \left(\frac{1}{2} \langle \Psi | \mathcal{Q} | \Psi \rangle + \frac{1}{3} \langle \Psi | \Psi * \Psi \rangle \right) \quad (2.1)$$

where

$$\mathcal{Q} = c_0 + \sum_{n>0} (-1)^n (c_{2n} + c_{-2n}) \quad (2.2)$$

The equation of motion is

$$\mathcal{Q}\Psi = -\Psi * \Psi \quad (2.3)$$

and the ansatz for nonperturbative solutions is in the factorized form

$$\Psi = \Psi_m \otimes \Psi_g \quad (2.4)$$

where Ψ_g and Ψ_m depend purely on ghost and matter degrees of freedom, respectively. Then eq.(2.3) splits into

$$\mathcal{Q}\Psi_g = -\Psi_g *_g \Psi_g \quad (2.5)$$

$$\Psi_m = \Psi_m *_m \Psi_m \quad (2.6)$$

where $*_g$ and $*_m$ refers to the star product involving only the ghost and matter part. The action for this type of solution becomes

$$\mathcal{S}(\Psi) = -\frac{1}{6g_0^2} \langle \Psi_g | \mathcal{Q} | \Psi_g \rangle \langle \Psi_m | \Psi_m \rangle \quad (2.7)$$

$\langle \Psi_m | \Psi_m \rangle$ is the ordinary inner product, $\langle \Psi_m |$ being the bpz conjugate of $|\Psi_m \rangle$ (see below).

It is well-known how to find solutions to (2.5). We pick one of them, but since the ghost part will not play a role we will simply ignore it in the following. We will instead be concerned with the matter part, eq.(2.6). The solutions are projectors of the $*_m$ algebra. The $*_m$ product is defined as follows

$${}_{123}\langle V_3 | \Psi_1 \rangle_1 | \Psi_2 \rangle_2 = {}_3 \langle \Psi_1 *_m \Psi_2 | \quad (2.8)$$

where the three strings vertex V_3 is

$$|V_3\rangle_{123} = \int d^{26}p_{(1)} d^{26}p_{(2)} d^{26}p_{(3)} \delta^{26}(p_{(1)} + p_{(2)} + p_{(3)}) \exp(-E) |0, p\rangle_{123} \quad (2.9)$$

with

$$E = \sum_{a,b=1}^3 \left(\frac{1}{2} \sum_{m,n \geq 1} \eta_{\mu\nu} a_m^{(a)\mu\dagger} V_{mn}^{ab} a_n^{(b)\nu\dagger} + \sum_{n \geq 1} \eta_{\mu\nu} p_{(a)}^\mu V_{0n}^{ab} a_n^{(b)\nu\dagger} + \frac{1}{2} \eta_{\mu\nu} p_{(a)}^\mu V_{00}^{ab} p_{(b)}^\nu \right) \quad (2.10)$$

Summation over the Lorentz indices $\mu, \nu = 0, \dots, 25$ is understood and η denotes the flat Lorentz metric. The operators $a_m^{(a)\mu}, a_m^{(a)\mu\dagger}$ denote the non-zero modes matter oscillators of the a -th string, which satisfy

$$[a_m^{(a)\mu}, a_n^{(b)\nu\dagger}] = \eta^{\mu\nu} \delta_{mn} \delta^{ab}, \quad m, n \geq 1 \quad (2.11)$$

$p_{(a)}$ is the momentum of the a -th string and $|0, p\rangle_{123} \equiv |p_{(1)}\rangle \otimes |p_{(2)}\rangle \otimes |p_{(3)}\rangle$ is the tensor product of the Fock vacuum states relative to the three strings with definite c.m. momentum. $|p_{(a)}\rangle$ is annihilated by the annihilation operators $a_m^{(a)\mu}$ ($m \geq 1$) and

it is eigenstate of the momentum operator $\hat{p}_{(a)}^\mu$ with eigenvalue $p_{(a)}^\mu$. The normalization is

$$\langle p_{(a)} | p'_{(b)} \rangle = \delta_{ab} \delta^{26}(p + p') \quad (2.12)$$

The symbols V_{nm}^{ab} , V_{0m}^{ab} , V_{00}^{ab} will denote the coefficients computed in [27, 28, 29, 30, 31]. We will use them in the notation of Appendix A and B of [10].

To complete the definition of the $*_m$ product we must specify the bpz conjugation properties of the oscillators

$$bpz(a_n^{(a)\mu}) = (-1)^{n+1} a_{-n}^{(a)\mu} \quad (2.13)$$

Let us now return to eq.(2.6). Its solutions are projectors of the $*_m$ algebra. The simplest one is the sliver. In this paper we will discuss a solution describing the decay of a D25-brane, represented by a sliver. We know that the sliver solution is in many regards too simple and too singular. However for our purposes in this paper it is better to avoid complications such as the addition of a background B field, [15], or dressing, [22, 23]. This will allow us to concentrate on the essential aspects of the problem, avoiding inessential formal complications. A more thorough treatment will be given elsewhere.

Let us recall the main points concerning the sliver solution. It is translationally invariant. As a consequence all momenta can be set to zero. The integration over the momenta can be dropped and the only surviving part in E will be the one involving V_{nm}^{ab} . The sliver is defined by

$$|\Xi\rangle = \mathcal{N} e^{-\frac{1}{2}a^\dagger S a^\dagger} |0\rangle, \quad a^\dagger S a^\dagger = \sum_{n,m=1}^{\infty} a_n^{\mu\dagger} S_{nm} a_m^{\nu\dagger} \eta_{\mu\nu} \quad (2.14)$$

This state satisfies eq.(2.6) provided the matrix S satisfies the equation

$$S = V^{11} + (V^{12}, V^{21})(1 - \Sigma \mathcal{V})^{-1} \Sigma \begin{pmatrix} V^{21} \\ V^{12} \end{pmatrix} \quad (2.15)$$

where

$$\Sigma = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{pmatrix}, \quad (2.16)$$

The proof of this fact is well-known. First one expresses eq.(2.16) in terms of the twisted matrices $X = CV^{11}$, $X_+ = CV^{12}$ and $X_- = CV^{21}$, together with $T = CS = SC$, where $C_{nm} = (-1)^n \delta_{nm}$. The matrices X, X_+, X_- are mutually commuting. Then, requiring T to commute with them as well, one can show that eq.(2.16) reduces to the algebraic equation

$$XT^2 - (1 + X)T + X = 0 \quad (2.17)$$

The sliver solution is

$$T = \frac{1}{2X} (1 + X - \sqrt{(1 + 3X)(1 - X)}) \quad (2.18)$$

The normalization constant \mathcal{N} is calculated to be

$$\mathcal{N} = (\text{Det}(1 - \Sigma\mathcal{V}))^{\frac{D}{2}} \quad (2.19)$$

The contribution of the sliver to the matter part of the action (see (2.7)) is given by

$$\langle \Xi | \Xi \rangle = \frac{\mathcal{N}^2}{(\det(1 - S^2))^{\frac{D}{2}}} \quad (2.20)$$

Both eq.(2.19) and (2.20) are ill-defined and need to be regularized. As anticipated we will not discuss this point here.

Now let us remark that there is another solution to (2.17), i.e. $1/T$. In fact (2.17) is invariant under the substitution $T \leftrightarrow 1/T$. $1/T$ is given by the RHS of eq.(2.18) with the $-$ sign replaced by the $+$ sign in front of the square root. We will call it the *inverse sliver*. This solution was previously discarded, [10], because of the bad asymptotic behaviour of the $1/T$ eigenvalues. However it is exactly this behaviour that will allow us, in the precise sense clarified in section 5, to find interesting time-dependent solutions⁵.

The sliver will be our reference solution in the following, but, in order to build our time-dependent solution, we will explicitly need another kind of solution, the lump [10]. The lump solution is engineered to represent a lower dimensional brane, therefore it will have transverse directions along which translational invariance is broken. Accordingly we split the three string vertex into the tensor product of the perpendicular part and the parallel part

$$|V_3\rangle = |V_{3,\perp}\rangle \otimes |V_{3,\parallel}\rangle \quad (2.21)$$

and the exponent E , accordingly, as $E = E_{\parallel} + E_{\perp}$. The parallel part is the same as in the sliver case while the perpendicular part is modified as follows. Following [10], we denote by x^{α}, p^{α} , $\alpha = 1, \dots, k$ the coordinates and momenta in the transverse directions and introduce the zero mode combinations

$$a_0^{(r)\alpha} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} - i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \quad a_0^{(r)\alpha\dagger} = \frac{1}{2}\sqrt{b}\hat{p}^{(r)\alpha} + i\frac{1}{\sqrt{b}}\hat{x}^{(r)\alpha}, \quad (2.22)$$

where $\hat{p}^{(r)\alpha}, \hat{x}^{(r)\alpha}$ are the zero momentum and position operator of the r -th string, and we have introduced the parameter b as in [10]. It follows

$$[a_0^{(r)\alpha}, a_0^{(s)\beta\dagger}] = \eta^{\alpha\beta}\delta^{rs} \quad (2.23)$$

Denoting by $|\Omega_b\rangle$ the oscillator vacuum ($a_0^{\alpha}|\Omega_b\rangle = 0$), the relation between the momentum basis and the oscillator basis is defined by

$$|\{p^{\alpha}\}\rangle_{123} = \left(\frac{b}{2\pi}\right)^{\frac{k}{4}} \exp \left[\sum_{r=1}^3 \left(-\frac{b}{4}p_{\alpha}^{(r)}\eta^{\alpha\beta}p_{\beta}^{(r)} + \sqrt{b}a_0^{(r)\alpha\dagger}p_{\alpha}^{(r)} - \frac{1}{2}a_0^{(r)\alpha\dagger}\eta_{\alpha\beta}a_0^{(r)\beta\dagger} \right) \right] |\Omega_b\rangle$$

⁵Notwithstanding the divergent behaviour of the eigenvalues it is perhaps possible to associate a definite meaning to the energy density of some of these solutions. They may be interesting as solutions of VSFT also without reference to time dependence.

Next we insert this equation inside E'_\perp and eliminate the momenta along the perpendicular directions by integrating them out. The overall result of this operation is that, while $|V_{3,\parallel}\rangle$ is the same as in the ordinary case,

$$|V_{3,\perp}\rangle' = K_k e^{-E'} |\Omega_b\rangle \quad (2.24)$$

with

$$K_k = \left(\frac{2\pi b^3}{\sqrt{3}(V_{00} + b/2)} \right)^{\frac{k}{4}}, \quad (2.25)$$

$$E' = \frac{1}{2} \sum_{r,s=1}^3 \sum_{M,N \geq 0} a_M^{(r)\alpha\dagger} V_{MN}'^{rs} a_N^{(s)\beta\dagger} \eta_{\alpha\beta} \quad (2.26)$$

where M, N denote the couple of indices $\{0, m\}$ and $\{0, n\}$, respectively. The coefficients $V_{MN}'^{rs}$ are given in Appendix B of [10]. The new Neumann coefficients matrices V'^{rs} satisfy the same relations as the V^{rs} ones. In particular one can introduce the matrices $X'^{rs} = CV'^{rs}$, where $C_{NM} = (-1)^N \delta_{NM}$, which turn out to commute with one another. All the relations of Appendix A hold with primed quantities. We can therefore repeat word by word the derivation of the sliver from eq.(2.14) through eq.(2.20). The new solution will have the form (2.14) with S along the parallel directions and S replaced by S' along the perpendicular ones. In turn S' is obtained as a solution to eq.(2.15) where all the quantities are replaced by primed ones. This amounts to solving eq.(2.17) with primed quantities. Therefore in the transverse directions S is replaced by S' , given by

$$S' = CT', \quad T' = \frac{1}{2X'} (1 + X' - \sqrt{(1 + 3X')(1 - X')}) \quad (2.27)$$

In a similar way we have to adapt the normalization and energy formulas (2.19,2.20).

Exactly as in the sliver case, we can consider the solution with T' replaced by $1/T'$. The same considerations hold as in that case.

3 Spectroscopy and diagonal representation

The diagonalization of the X matrix was carried out in [24], while the same analysis for X' was accomplished in [26] and [25]. Here, for later use, we summarize the results of these references. The eigenvalues of $X = X^{11}, X_+ = X^{12}, X_- = X^{21}$ and T are given, respectively, by

$$\mu^{rs}(k) = \frac{1 - 2\delta_{r,s} + e^{\frac{\pi k}{2}} \delta_{r+1,s} + e^{-\frac{\pi k}{2}} \delta_{r,s+1}}{1 + 2 \cosh \frac{\pi k}{2}} \quad (3.1)$$

$$t(k) = -e^{-\frac{\pi|k|}{2}} \quad (3.2)$$

where $-\infty < k < \infty$. The generating function for the eigenvectors is

$$f^{(k)}(z) = \sum_{n=1}^{\infty} v_n^{(k)} \frac{z^n}{\sqrt{n}} = \frac{1}{k} (1 - e^{-k \arctan z}) \quad (3.3)$$

The completeness and orthonormality equations for the eigenfunctions are as follows

$$\sum_{n=1}^{\infty} v_n^{(k)} v_n^{(k')} = \mathcal{N}(k) \delta(k - k'), \quad \mathcal{N}(k) = \frac{2}{k} \sinh \frac{\pi k}{2}, \quad \int_{-\infty}^{\infty} dk \frac{v_n^{(k)} v_m^{(k)}}{\mathcal{N}(k)} = \delta_{nm} \quad (3.4)$$

The spectrum of X is continuous and lies in the interval $[-1/3, 0)$. It is doubly degenerate except at $-\frac{1}{3}$. The continuous spectrum of X' lies in the same interval, but X' in addition has a discrete spectrum. To describe it we follow [25]. We consider the decomposition

$$X'^{rs} = \frac{1}{3} (1 + \alpha^{s-r} CU' + \alpha^{r-s} U'C) \quad (3.5)$$

where $\alpha = e^{\frac{2\pi i}{3}}$. It is convenient to express everything in terms of CU' eigenvalues and eigenvectors (see Appendix B). The discrete eigenvalues are denoted by ξ and $\bar{\xi}$. Since CU' is unitary they lie on the unit circle. They are more effectively represented via the parameter η , (B.5), which in turn is connected to the parameter b (B.7). To each value of b there corresponds a couple of values of η with opposite sign (except for $b = 0$ which implies $\eta = 0$).

The eigenvectors corresponding to the continuous spectrum are $V_N^{(k)}$ ($-\infty < k < \infty$), while the eigenvectors of the discrete spectrum are denoted by $V_N^{(\xi)}$ and $V_N^{(\bar{\xi})}$. They form a complete basis. They will be normalized so that the completeness relation takes the form

$$\int_{-\infty}^{\infty} dk V_N^{(k)} V_M^{(k)} + V_N^{(\xi)} V_M^{(\xi)} + V_N^{(\bar{\xi})} V_M^{(\bar{\xi})} = \delta_{NM} \quad (3.6)$$

It has become familiar and very useful to expand all the relevant quantities in VSFT by means of this basis. To this end we define

$$\begin{aligned} a_k &= \sum_{N=0}^{\infty} V_N^{(k)} a_N, & a_{\xi} &= \sum_{N=0}^{\infty} V_N^{(\xi)} a_N, & a_{\bar{\xi}} &= \sum_{N=0}^{\infty} V_N^{(\bar{\xi})} a_N \\ a_N &= \int_{-\infty}^{\infty} dk V_N^{(k)} a_k + V_N^{(\xi)} a_{\xi} + V_N^{(\bar{\xi})} a_{\bar{\xi}} \end{aligned} \quad (3.7)$$

and introduce the even and odd twist combinations

$$e_k = \frac{a_k + Ca_k}{\sqrt{2}}, \quad e_{\eta} = \frac{a_{\xi} + Ca_{\xi}}{\sqrt{2}}, \quad o_k = \frac{a_k - Ca_k}{i\sqrt{2}}, \quad o_{\eta} = \frac{a_{\xi} - Ca_{\xi}}{i\sqrt{2}}, \quad (3.8)$$

The commutation relations among them are

$$[e_k, e_{k'}^\dagger] = \delta(k - k'), \quad [e_\eta, e_\eta^\dagger] = 1, \quad [o_k, o_{k'}^\dagger] = \delta(k - k'), \quad [o_\eta, o_\eta^\dagger] = 1, \quad (3.9)$$

while all the other commutators vanish. The twist properties are defined by

$$Ca_k = a_{-k}, \quad Ca_\xi = a_{\bar{\xi}},$$

Using these combinations the three–strings vertex can be cast in diagonal form and, for instance, the exponent of the conventional lump state can be written

$$\begin{aligned} a^\dagger S' a^\dagger &= \int_{-\infty}^{\infty} dk t(k) (a_k^\dagger, Ca_k^\dagger) + 2 t_\xi (a_\xi^\dagger, Ca_\xi^\dagger) \\ &= \frac{1}{2} \int_{-\infty}^{\infty} dk t(k) (e_k^\dagger e_k^\dagger + o_k^\dagger o_k^\dagger) + t_\eta (e_\eta^\dagger e_\eta^\dagger + o_\eta^\dagger o_\eta^\dagger) \end{aligned} \quad (3.10)$$

where $t_\eta \equiv t_\xi = e^{-|\eta|}$. The unconventional lump is obtained by replacing t_η with its inverse $e^{|\eta|}$.

In the sequel we need the behaviour of the eigenvectors when $b \rightarrow 0$ and when $b \rightarrow \infty$. Near $b = 0$ we have

$$\begin{aligned} b &\approx 0, & \eta &\approx 0, & \xi &\approx 1 \\ V_0^{(\xi)} &= \frac{1}{\sqrt{2}} + \mathcal{O}(\eta), & V_n^{(\xi)} &= \mathcal{O}(\eta^2) \end{aligned} \quad (3.11)$$

The same behaviour holds for the $V^{(\bar{\xi})}$ basis.

When $b \rightarrow \infty$ we have instead

$$\begin{aligned} b &\rightarrow \infty, & b &\approx 4 \log \eta, & \xi &\approx -e^{\frac{\pi i}{3}} \\ V_0^{(\xi)} &\approx e^{-\frac{\eta}{2}} \sqrt{2 \eta \log \eta}, & V_n^{(\xi)} &\sim e^{-\frac{\eta}{2}} \sqrt{\eta} \end{aligned} \quad (3.12)$$

and the same for $V^{(\bar{\xi})}$.

These asymptotic behaviours will be used to evaluate matrix elements such as (4.8). In this regard they are completely reliable (and, in any case, backed up by numerical evidence). If we consider instead the corresponding asymptotic expansions for the $V^{(k)}$ basis, we have to be more careful. The point is that the expression $(V_0^{(k)})^2$, see (B.11), would superficially seem to vanish in the limit $b \rightarrow \infty$, but it is in fact a representation of the Dirac delta function $\delta(k)$, see Appendix D. Therefore the result of taking the $b \rightarrow \infty$ limit in an integral containing $(V_0^{(k)})^2$ is to concentrate it at the point $k = 0$. This renders the generating function (B.10) very singular and, consequently, such integrals as $\int dk V_n^{(k)} V_m^{(k)} f(k)$ must be handled with care. As for the limit of the continuous basis when $b \rightarrow 0$, one can see that $V_0^{(k)} \rightarrow 0$, while the other eigenfunctions have a nonvanishing finite limit.

4 Time dependent solutions: dead ends

In order to appreciate the very nature of the problem of finding time-localized VSFT solutions, let us examine first some obvious attempts and learn from their failure. The first thing that comes to one's mind is to start from a lump with one transverse space direction (therefore it represents a D24-brane) and inverse-Wick-rotate it. One such solution has been introduced above, see section 2 from eq.(2.21) through eq.(2.27). For simplicity we denote the transverse direction coordinate, momentum and oscillators simply by x, p and a_N . The solution is written as follows:

$$\begin{aligned} |\Psi'\rangle &= |\Xi\rangle_{25} \otimes |\Lambda'\rangle \\ |\Lambda'\rangle &= \mathcal{N}' \exp \left[-\frac{1}{2} \sum_{N,M \geq 0} a_N^\dagger S'_{NM} a_M^\dagger \right] |\Omega_b\rangle \end{aligned} \quad (4.1)$$

where $|\Xi\rangle_{25}$ is the usual sliver along the longitudinal 25 directions and

$$\mathcal{N}' = \sqrt{3} \frac{V_{00} + \frac{b}{2}}{(2\pi b^3)^{\frac{1}{4}}} \sqrt{\det(1 - X') \det(1 + T')} \quad (4.2)$$

In order to study the space profile of this solution in the transverse direction we contract it with the x_0 -coordinate eigenstate

$$|x_0\rangle = \left(\frac{2}{b\pi} \right)^{\frac{1}{4}} \exp \left[-\frac{1}{b} x_0^2 - \frac{2}{\sqrt{b}} i a_0^\dagger x_0 + \frac{1}{2} (a_0^\dagger)^2 \right] |\Omega_b\rangle \quad (4.3)$$

The result is

$$\langle x_0 | \Lambda' \rangle = \left(\frac{2}{b\pi} \right)^{\frac{1}{4}} \frac{\mathcal{N}'}{\sqrt{1+s'}} \exp \left[\frac{1}{b} \frac{s'-1}{s'+1} x_0^2 - \frac{2i}{\sqrt{b}} \frac{x_0 f_0}{1+s'} - \frac{1}{2} a^\dagger W' a^\dagger \right] \quad (4.4)$$

where the condensed notation means

$$f_0 = \sum_{n=1} S'_{0n} a_n^\dagger, \quad a^\dagger W' a^\dagger = \sum_{n,m=1} a_n^\dagger W'_{nm} a_m^\dagger, \quad W'_{nm} = S'_{nm} - \frac{S'_{0n} S'_{0m}}{1+s'} \quad (4.5)$$

and

$$s' = S'_{00} \quad (4.6)$$

After an inverse Wick-rotation $x_0 \rightarrow ix_0, a_n^\dagger \rightarrow ia_n^\dagger$ (4.4) becomes

$$\langle x_0 | \Lambda' \rangle = \left(\frac{2}{b\pi} \right)^{\frac{1}{4}} \frac{\mathcal{N}'}{\sqrt{1+s'}} \exp \left[\frac{1}{b} \frac{1-s'}{1+s'} x_0^2 + \frac{2i}{\sqrt{b}} \frac{x_0 f_0}{1+s'} + \frac{1}{2} a^\dagger W' a^\dagger \right] \quad (4.7)$$

We are interested in solutions localized in time. The second term in the exponent gives rise to time oscillations. Only the first term can guarantee time localization.

Precisely this happens when $|s'| > 1$. However such a condition can never be achieved within the present scheme in which ordinary lump solutions are utilized. In fact it is possible to show that for such solutions $|s'| \leq 1$. Therefore with the simple-minded scheme considered so far it is impossible to achieve time localization (in this regard our negative conclusion is similar to [21]; as for the case $b \rightarrow 0$, see below).

Let us see this in more detail by showing that $|s'| \leq 1$. Using the basis of the previous section we can write

$$s' \equiv S'_{00} = \int_{-\infty}^{\infty} dk V_0^{(k)} (-e^{-\frac{\pi|k|}{2}}) V_0^{(k)} + V_0^{(\xi)} e^{-|\eta|} V_0^{(\xi)} + V_0^{(\bar{\xi})} e^{-|\eta|} V_0^{(\bar{\xi})} \quad (4.8)$$

Using (B.11), one can see that the first term in the RHS does not contribute in the limit $b \rightarrow 0$ (i.e. $\eta \rightarrow 0$) and using the approximants (3.11) we immediately see that the remaining two terms add up to 1. Therefore when $b \rightarrow 0$, $s' \rightarrow 1$. Viceversa, in the limit $b \rightarrow \infty$, using (3.12) we see that the last two terms in the RHS of (4.8) do not contribute, while the first term contribute exactly -1 . This can be also shown numerically or with the alternative analytical method of Appendix C. For generic values of b we cannot calculate s' analytically but it is easy to evaluate it numerically and to show that it is a monotonically decreasing function of b for $0 \leq b < \infty$. This in turn implies that the quantity $\frac{1-s'}{1+s'}$ is always *positive* (see figure 1).

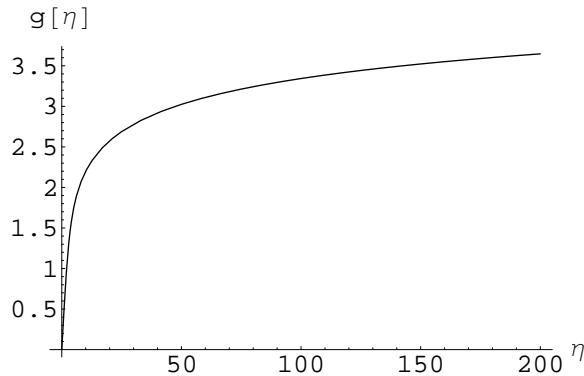


Figure 1: The quantity $g[\eta] = \frac{1-s'}{1+s'}$ as a function of η

Our conclusion is therefore that we cannot obtain a time-localized solution by inverse-Wick-rotating an ordinary lump solution. We will show elsewhere that also adding a constant B -field does not change this negative conclusion. Some drastic change has to be made in order to produce a localized time-dependent solution.

5 A Rolling Tachyon–like Solution

It is not hard to realize that if we were to replace $e^{-|\eta|}$ with $e^{|\eta|}$ in eq.(4.8) we would reverse the conclusion at the end of the previous section. In fact, see below, we would have $|s'| \geq 1$. In this section we wish to exploit this possibility. In section 2 we have seen that if in the lump solution we replace T' by $1/T'$, formally, we still have a projector. Motivated by this fact we define an unconventional lump, by replacing $|\check{\Lambda}'\rangle$ in (4.1) with

$$|\check{\Lambda}'\rangle = \check{\mathcal{N}}' \exp \left(-\frac{1}{2} a^\dagger C \check{T}' a^\dagger \right) |\Omega_b\rangle \quad (5.1)$$

where

$$\check{T}'_{NM} = - \int_{-\infty}^{\infty} dk V_N^{(k)} V_M^{(k)} \exp \left(-\frac{\pi|k|}{2} \right) + \left(V_N^{(\xi)} V_M^{(\xi)} + V_N^{(\bar{\xi})} V_M^{(\bar{\xi})} \right) \exp |\eta| \quad (5.2)$$

Due to the fact that the star product is split into eigenspaces of the Neumann coefficients X' , X'_+ , X'_- , the projector equation split accordingly into the continuous and discrete spectrum part. Therefore we are guaranteed that (5.1) is again a projector, as one can on the other hand easily verify by direct calculation. This is the solution we propose.

Before we proceed with our analysis we would like to clarify a basic question about the solution we have just put forward. Passing from a squeezed state solution with a matrix T' to another characterized by the inverse matrix $1/T'$ may lead in general to unacceptable features of the state, such as divergent terms in the oscillator basis. However in the case at hand, in which one inverts only the discrete spectrum, such unpleasant aspects disappear. First of all the matrix \check{T}' is well defined both in the oscillator and in the diagonal basis. Second, such expression as $\sqrt{\det(1 - \check{T}')}}$ are well-defined. This is due to the fact that, if we are allowed to factorize the discrete and continuous spectrum contribution, the former can be written as $\det(1 - \check{T}')^{(d)} = (1 - \exp|\eta|)^2$, so that the possible dangerous – sign under the square root disappears due to the double multiplicity of the discrete eigenvalue. Third, the energy density of the (Euclidean) solution (5.1) equals the energy density of the ordinary lump. In fact, using the formulas of [10], the ratio between the energy densities of the two solutions reduces to

$$\sqrt{\frac{\det(1 + \check{T}')}{\det(1 - \check{T}')}} / \sqrt{\frac{\det(1 + T')}{\det(1 - T')}} = \sqrt{\frac{(1 + e^{|\eta|})^2}{(1 - e^{|\eta|})^2}} / \sqrt{\frac{(1 + e^{-|\eta|})^2}{(1 - e^{-|\eta|})^2}} = 1 \quad (5.3)$$

after factorization of the discrete and continuous parts of the spectrum.

After these important remarks it remains for us to show that this solution has the appropriate features to represent a rolling tachyon solution. To see if this is true we have to represent it in a more explicit way. In particular we have to extract the explicit time dependence (better, the space dependence and then inverse–Wick–rotate it). To

do so, we have to choose a (coordinate) basis on which to project (5.1). There seem to be two distinguished ways to make this choice. We will work them out explicitly and then discuss them.

To start with let us define the following coordinate and momentum operator, given by the twist even and twist odd parts of the discrete spectrum,

$$\hat{x}_\eta = \frac{i}{\sqrt{2}}(e_\eta - e_\eta^\dagger) \quad (5.4)$$

$$\hat{y}_\eta = \frac{i}{\sqrt{2}}(o_\eta - o_\eta^\dagger) \quad (5.5)$$

The eigenstates of the coordinate \hat{x}_η are given by

$$\begin{aligned} |x\rangle &= \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2}x^2 - \sqrt{2}ie_\eta^\dagger x + \frac{1}{2}e_\eta^\dagger e_\eta\right) |\Omega_{\eta_e}\rangle, \\ e_\eta |\Omega_{\eta_e}\rangle &= 0 \\ \hat{x}_\eta |x\rangle &= x|x\rangle \end{aligned} \quad (5.6)$$

Correspondingly the eigenstates of the momentum \hat{y}_η are

$$\begin{aligned} |y\rangle &= \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2}y^2 - \sqrt{2}io_\eta^\dagger y + \frac{1}{2}o_\eta^\dagger o_\eta\right) |\Omega_{\eta_o}\rangle, \\ o_\eta |\Omega_{\eta_o}\rangle &= 0 \\ \hat{y}_\eta |y\rangle &= y|y\rangle \end{aligned} \quad (5.7)$$

In order to make the x, y dependence explicit we project our solution (5.1) into the position/momentum eigenstates (5.6, 5.7). Using standard results⁶ we get

$$\langle x, y | \check{\Lambda}' \rangle = \frac{1}{\pi(1 + e^{|\eta|})} \exp\left(\frac{e^{|\eta|} - 1}{e^{|\eta|} + 1}(x^2 + y^2)\right) |\check{\Lambda}'_c\rangle \quad (5.8)$$

The state $|\check{\Lambda}'_c\rangle$ is given by (5.1), but with only oscillators from the continuous spectrum, as the contribution of the discrete spectrum is now contained in the prefactor at the rhs of (5.8). Now we perform the inverse Wick rotation $x \rightarrow ix$, $y \rightarrow -iy$ to recover the Lorentz signature, and obtain

$$|\check{\Lambda}'(x; y)\rangle = \frac{1}{\pi(1 + e^{|\eta|})} \exp\left(-\frac{e^{|\eta|} - 1}{e^{|\eta|} + 1}(x^2 + y^2)\right) |\check{\Lambda}'_c\rangle^{(Wick)} \quad (5.9)$$

It is evident that for every value of η the solution is localized in the x -time coordinate. The extra coordinate y is related to internal twist odd degrees of freedom and can be interpreted as a free parameter of the representation (5.9). This solution also contains the free parameter η which is nothing but a reparametrization of b , through (B.7). Therefore it is characterized by two free parameters.

⁶Here we are assuming that the vacuum factorizes into $|\Omega_{\eta_e}\rangle \otimes |\Omega_{\eta_o}\rangle \otimes |\Omega_c\rangle$ where the latter factor represents the vacuum with respect to the continuous oscillator component.

The ‘time’ x is not the ordinary time, i.e. the time coupled to the flat open string metric and related to the string center of mass. We will see later on a possible interpretation for x . Now, let us turn to the ordinary (open string) time, i.e. the time defined by the center of mass of the string and analyze the corresponding time profile. Despite the fact that this coordinate is not diagonal for the $*$ -product we can still have complete control on the profile along it. The center of mass position operator is given by

$$\hat{x}_0 = \frac{i}{\sqrt{b}}(a_0 - a_0^\dagger) \quad (5.10)$$

The center of mass position eigenstate is

$$|x_0\rangle = \left(\frac{2}{b\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{1}{b}x_0x_0 - \frac{2}{\sqrt{b}}ia_0^\dagger x_0 + \frac{1}{2}a_0^\dagger a_0^\dagger\right) |\Omega_b\rangle \quad (5.11)$$

Let us compute the center of mass time profile. After inverse-Wick-rotating it, it turns out to be

$$\begin{aligned} |\check{\Lambda}'(x_0)\rangle &= \langle x_0|\check{\Lambda}'\rangle = \\ \left(\frac{2}{b\pi}\right)^{\frac{1}{4}} \frac{\mathcal{N}'}{\sqrt{1+\check{T}'_{00}}} &\exp\left(\frac{1}{b}\frac{1-\check{T}'_{00}}{1+\check{T}'_{00}}x_0^2 + \frac{2i}{\sqrt{b}(1+\check{T}'_{00})}x_0\check{T}'_{0n}a_n^\dagger + \frac{1}{2}a_n^\dagger W'_{nm}a_m^\dagger\right) |\Omega_b\rangle \end{aligned} \quad (5.12)$$

$$W'_{nm} = \check{S}'_{nm} - \frac{1}{1+\check{T}'_{00}}\check{S}'_{0n}\check{S}'_{0m} \quad (5.13)$$

The quantities \check{S}'_{0n} and \check{S}'_{nm} can be computed in the diagonal basis

$$\begin{aligned} \check{S}'_{0n} &= \check{T}'_{0n} \\ &= (1+(-1)^n)\left(-\int_0^\infty V_0^{(k)}V_n^{(k)}\exp\left(-\frac{\pi k}{2}\right) + V_0^{(\xi)}V_n^{(\xi)}\exp|\eta|\right) \end{aligned} \quad (5.14)$$

$$\begin{aligned} \check{S}'_{nm} &= (-1)^n\check{T}'_{nm} = \\ &= ((-1)^n+(-1)^m)\left(-\int_0^\infty V_n^{(k)}V_m^{(k)}\exp\left(-\frac{\pi k}{2}\right) + V_n^{(\xi)}V_m^{(\xi)}\exp|\eta|\right) \end{aligned} \quad (5.15)$$

It is evident that the leading time dependence in (5.12), for large x_0 , is contained in $\exp\left(\frac{1}{b}\frac{1-\check{T}'_{00}}{1+\check{T}'_{00}}x_0^2\right)$. The number \check{T}'_{00} is $b(\eta)$ -dependent and can be computed via

$$\check{T}'_{00}(\eta) = -2\int_0^\infty dk \left(V_0^{(k)}(b(\eta))\right)^2 \exp\left(-\frac{\pi k}{2}\right) + 2(V_0^{(\xi)})^2 \exp|\eta| \quad (5.16)$$

This is the crucial quantity as far as the time profile is concerned. An analytic evaluation of it is beyond our reach. However we will later show that

$$\lim_{\eta \rightarrow 0} \check{T}'_{00} = 1 \quad (5.17)$$

$$\lim_{\eta \rightarrow \infty} \check{T}'_{00} = \infty \quad (5.18)$$

A numerical analysis shows that this quantity is a function monotonically increasing with η within such limits. This means that the quantity $\frac{1-\check{T}'_{00}}{1+\check{T}'_{00}}$ is always *negative* (it lies in the interval $(-1, 0)$, see figure 2) and so the profile is always localized in the center of mass time, except in the extreme case $\eta \rightarrow 0$, which corresponds to the tensionless limit.

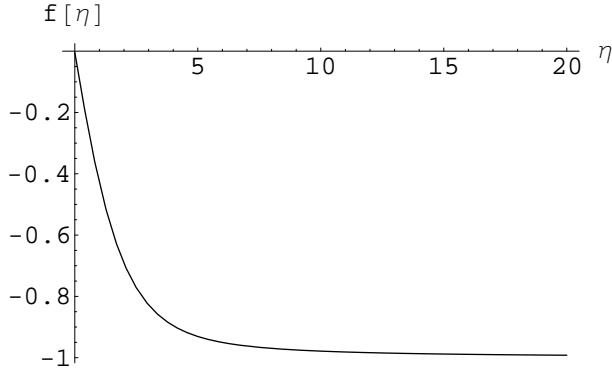


Figure 2: The quantity $f[\eta] = \frac{1-\check{T}'_{00}}{1+\check{T}'_{00}}$ as a function of η

This has to be compared with the usual lump solution (see previous section) for which the corresponding quantity is always positive and takes values in the interval $(0, \infty)$, allowing for localized *space* profiles but divergent along a timelike direction.

For reasons that will become clear in the next section, we extract also the free parameter y dependence, by projecting onto the corresponding twist-odd eigenstate (5.7). This operation can be done before or after the projection along the center of mass coordinate and does not interfere with it because \hat{y} does not contain the zero mode. We will therefore consider the following representation of our solution (inverse Wick rotation is again understood)

$$|\Lambda'(x_0, y)\rangle = \langle x_0, y | \check{\Lambda}' \rangle = \left(\frac{2}{b\pi} \right)^{\frac{1}{4}} \frac{\check{\mathcal{N}'}}{\sqrt{2\pi(1+e^{|\eta|})}} \exp \left(\frac{1-e^{|\eta|}}{1+e^{|\eta|}} y^2 \right) \langle 0 | \exp \left(\frac{1}{b} \frac{1-\check{T}'_{00}}{1+\check{T}'_{00}} x_0^2 + \frac{2i}{\sqrt{b}(1+\check{T}'_{00})} x_0 \check{T}'_{0n} a_n^\dagger - \frac{1}{2} a_n^\dagger W''_{nm} a_m^\dagger \right) | 0 \rangle \quad (5.19)$$

The quantities \check{T}'_{00} and \check{T}'_{0n} are the same as in (5.16, 5.14) since the momentum \hat{y}_n is twist-odd. Some changes occur in W''_{nm}

$$W''_{nm} = \check{S}''_{nm} - \frac{1}{1+\check{T}'_{00}} \check{S}'_{0n} \check{S}'_{0m} \quad (5.20)$$

$$\check{S}''_{nm} = ((-1)^n + (-1)^m) \left(- \int_0^\infty dk V_n^{(k)} V_m^{(k)} \exp \left(-\frac{\pi k}{2} \right) + V_n^{(\eta)} V_m^{(\eta)} \exp |\eta| \right) \quad n, m \text{ even}$$

$$= -((-1)^n + (-1)^m) \int_0^\infty dk V_n^{(k)} V_m^{(k)} \exp\left(-\frac{\pi k}{2}\right) \quad n, m \text{ odd}$$

Note that \check{S}_{nm}'' gets contribution only from the twist-even part of the discrete spectrum.

In conclusion (5.19) provides the solution we were looking for. It represents a solution localized in x_0 , with the desired profile. It depends on two free parameters y and η (or b). These are all positive features. But let us start making a closer comparison with the rolling tachyon solution (such a comparison is made with the representation (5.19)). This can be done by considering the limit $b \rightarrow \infty$, which can be derived from the eqs.(3.12). $b \rightarrow \infty$ means $\eta \rightarrow \infty$ (for simplicity from now on we take η positive) and

$$\check{T}'_{00} \approx 2\eta \log \eta \left(1 - \frac{\log(2\pi)}{\log \eta} + \dots\right) \quad (5.21)$$

where dots denote higher order terms. Therefore we see that in this limit any time dependence in (5.19) disappears. Moreover, anticipating a result of the following section, we also have that $W''_{nm} \rightarrow S_{nm}$. In other words, in the limit $b \rightarrow \infty$ we obtain a static solution corresponding to the initial sliver. From this we understand that the parameter $1/b$, for large b , plays a role similar to Sen's parameter $\tilde{\lambda}$ near 0⁷. A second remark concerns the limit $y \rightarrow \infty$. In this case the first exponential factor in the RHS of (5.19) suppresses everything, so that the limit is the 0 state. In other words, we can consider this value of the parameter y as identifying the (relatively) stable vacuum state.

Although the rolling tachyon naturally compares with (5.19) rather than with (5.9), it is instructive to repeat something similar with the latter. Let us stress once more that both (5.19) and (5.9) represent the same solution, but in different bases, in particular with two different times: one, x_0 , is the open string center of mass time, the other, x , is related to the discrete spectrum. In the (5.9) case a parameter like b is missing. But this is something that is simply not customary and can be easily remedied. We can in fact introduce a parameter b_e in (5.4,5.5), just replacing $\sqrt{2}$ with $\sqrt{b_e}$ in those equations. Then (5.9) would become

$$|\check{\Lambda}'(x; y)\rangle = \frac{1}{b_e \pi (1 + e^\eta)} \exp\left(-\frac{1}{b_e} \frac{e^\eta - 1}{e^\eta + 1} (x^2 + y^2)\right) |\check{\Lambda}'_c\rangle^{(Wick)} \quad (5.22)$$

and we could repeat the same argument as above and reach the same conclusion, except that in this case we have to take $b_e \rightarrow \infty$ as well as $\eta \rightarrow \infty$. The limit $y \rightarrow \infty$ plays the same role as in the (5.19) representation.

In the next section we will study the solution (5.1) in a regime we are more familiar with, the low energy regime $\alpha' \rightarrow 0$, and in the other extreme regime, $\alpha' \rightarrow \infty$,

⁷We recall that Sen's rolling tachyon solution depends on the parameter $\tilde{\lambda}$, which appears in the $\tilde{\lambda} \int_{\partial D} dt \cosh X^0(t)$ -deformed BCFT.

in which the solution considerably simplifies and an analytic treatment is possible. What we would like to see more closely is whether, for sufficiently small values of the parameters, the solution at time 0 is close enough to the sliver configuration (2.14), whose decay the solution is expected to describe.

6 Low energy and tensionless limits

As reviewed in appendix C, the low energy limit is obtained by performing an $\epsilon \rightarrow 0$ limit on the quantities that depend on the Neumann coefficients of the three strings vertex. ϵ is a dimensionless parameter that represents the smallness of α' , [11]. As it happens, in all the expansions we consider, the parameters ϵ and b only appear through the ratio ϵ/b . Therefore, formally, the expansions for small ϵ/b are the same as the expansions for large b , i.e. $\eta \rightarrow \infty$. Therefore, in this section, when we consider the expansion in η near ∞ we really mean the expansion for ϵ/b small (i.e. ϵ small and b finite). A different attitude is required by the ‘external’ states like (4.3). There the rescaling of x_0 would lead to the replacement $b \rightarrow b\epsilon$. In this case we absorb ϵ into x_0 and keep b finite. In conclusion, throughout the analysis of the low energy limit, b should be considered as a finite free parameter.

Let us analyze in detail what is the limit of the various quantities appearing in (5.19). First of all we have

$$\lim_{\eta \rightarrow \infty} \frac{1 - \check{T}'_{00}}{1 + \check{T}'_{00}} = -1 \quad (6.1)$$

This follows from (3.12) and from the discussion at the end of section 3, in particular from the property of $(V_0^{(k)})^2$ of approximating $\delta(k)$ in the limit $b \rightarrow \infty$, which implies that $\check{T}'_{00} \rightarrow \infty$ in the same limit. For the oscillating term we have

$$\lim_{\eta \rightarrow \infty} \frac{\check{T}'_{0n}}{1 + \check{T}'_{00}} = \lim_{\eta \rightarrow \infty} \frac{1}{\sqrt{2 \log \eta}} = 0 \quad (6.2)$$

To evaluate this limit one must evaluate \check{T}'_{0n} . This in turn requires knowing the asymptotic expansion of the basis $V_n^{(k)}$ for $\eta \rightarrow \infty$. This is done in Appendix D. A numerical approximation confirms the above result.

Thus, in the limit, the oscillating part completely decouples from the time dependent part. It remains for us to consider the limit of the quadratic form W''_{nm} , (5.20). When n, m are odd there are no contributions from the discrete spectrum, since the contraction with $\langle y |$ has eliminated them.

$$W''_{2n-1, 2m-1} = \check{S}_{2n-1, 2m-1}^{'(c)}, \quad (6.3)$$

When n, m are even we have, on the contrary, potentially dangerous terms because there are divergent contributions arising from the discrete spectrum. The latter have

to be carefully evaluated.

$$\begin{aligned}
W_{2n,2m}''^{(d)} &= 2V_{2n}^{(\xi)}V_{2m}^{(\xi)} \left(e^\eta - \frac{2(V_0^{(\xi)})^2 e^{2\eta}}{2(V_0^{(\xi)})^2 e^\eta + \mathcal{O}(\frac{e^{-\eta}}{\eta \log \eta})} \right) \\
&= 2V_{2n}^{(\xi)}V_{2m}^{(\xi)} \mathcal{O}(\frac{e^{-\eta}}{\eta \log \eta}) \approx \mathcal{O}(\frac{1}{\log \eta})
\end{aligned} \tag{6.4}$$

We see that the potentially divergent contributions arising from the discrete spectrum exactly cancel when $\eta \rightarrow \infty$. Therefore, as far as $W_{2n,2m}''^{(d)}$ is concerned, we are left only with the contribution from the continuous spectrum. Of the two pieces that contribute to $W_{2n,2m}''^{(c)}$, see eq.(5.20) only the first survives in the limit $\eta \rightarrow \infty$, the second vanishes for the usual reasons. Therefore we can conclude that

$$W_{nm}'' = \check{S}_{nm}^{(c)} + \dots$$

where dots denote subleading corrections of order at least $1/\log \eta$. At this stage we can do the calculation directly as in Appendix D, or we can resort to an indirect argument by noticing that $\check{S}_{nm}^{(c)}$ approaches $S_{nm}^{(c)}$ in the same limit, because the discrete spectrum contribution to the latter vanishes, and then use the results of Appendix C. In both cases we conclude that

$$W_{nm}'' = S_{nm} + \mathcal{O}(\epsilon/b) \tag{6.5}$$

Going back to equation (5.19) we see that, modulo a normalization factor, we obtain

$$\lim_{\alpha' \rightarrow 0} |\check{\Lambda}'(x_0, y)\rangle = \check{\mathcal{N}}'(y) e^{-\frac{x_0^2}{b}} |\Xi\rangle \tag{6.6}$$

where $|\Xi\rangle$ is the zero momentum sliver state. This result can be phrased as follows: in the low energy limit the solution takes the form of a time-Gaussian multiplying a sliver, the subleading terms being proportional to ϵ/b , eq.(6.5).

To end this section let us briefly consider the opposite limit, that is $\alpha' \rightarrow \infty$ (tensionless limit). As in the previous case this is formally achieved by taking the $\eta \rightarrow 0$ limit in all the quantities which are related to the Neumann coefficients, but leaving b as a free parameter. This limit is well defined. Using the results of appendix C we get

$$\lim_{\eta \rightarrow 0} \frac{1 - \check{T}'_{00}}{1 + \check{T}'_{00}} = 0 \tag{6.7}$$

The oscillating term in (5.19) vanishes as well. This result implies that the Gaussian representing time dependence in (5.19) is actually completely flat: time dependence has disappeared! We believe this to be related to the fact that all strings modes become massless in this limit [32], so there are no modes to decay into. It is easy to see that the only non vanishing term in the exponent of (5.19) is the quadratic

part which gets contribution only from the continuous spectrum (on the contrary of the $\eta \rightarrow \infty$ limit the discrete eigenvector has only the 0-component, while the higher components disappear like positive powers of $1/\eta$). We remark that in the tensionless limit the center of mass time and the x time are identified.

7 Discussion

In the last two sections we have shown that by inverting the discrete part of the spectrum we obtain a definite (unconventional) lump solution which, after inverse Wick-rotation, gives rise to a time-localized state with many properties characteristic of the rolling tachyon solution. In the course of our exposition we have left aside some loose ends which we would like now to tie up or at least comment upon.

The first comment concerns normalization of the states we have come across. We have written down throughout normalization factors in quite a formal way. We have already recalled the fact that the sliver state and the lump state have a vanishing normalization, but we believe these problems have to be kept separated from the normalization of our time dependent solution. As a matter of fact a normalization problem appears only for the representation (5.19) and in the low energy limit, for the coefficient \check{N}' in (6.6) diverges exponentially for $\eta \rightarrow \infty$ once all the contributions are taken into account (this problem does not arise for the other representation (5.9)). We remark however that, as was noticed in the discussion after eq.(5.1), the energy density of the corresponding Euclidean solution is well-defined (once the conventional lump energy density is). Therefore the exploding normalization can only be an artifact of the representation. It means that we have to use the parameters of the state to regulate the normalization, although it is not clear a priori what is the right way to do it. A possibility is to use the factor $\exp\left(\frac{1-e^{|\eta|}}{1+e^{|\eta|}}y^2\right)$ in (5.19). Since this vanishes for y large, we can view y as a suitable function of η as $\eta \rightarrow \infty$. This can settle the problem. Other possibilities are connected to dressing, [22, 23].

We would like to add a comment concerning the meaning of our solution (5.1) before inverse Wick-rotation. As we have noticed, its profile is an inverted Gaussian that explodes at infinity. This suggests that we can interpret it as a D-brane located at infinity in the transverse direction, that is at infinite imaginary time. One could speculate this to be linked to the D-branes at imaginary times referred to in [1, 4].

Another important question is the number of parameters. Our solution depends on two parameters y and b . One may wonder why we extracted the y dependence from 5.1. This is indeed not a choice but a constraint. Had we not done it, we would have found a different formula (5.20) in which also the n, m odd part of S''_{nm} would have taken a contribution from the discrete spectrum (exactly as the n, m even part). However in the odd-odd part no such cancelation (6.4) as in the even-even part occurs and we would find badly divergent coefficients in W' . We gather that y is a genuine free parameter of the time-dependent state. What about b ? It was

argued in [10] that this parameter represents a gauge degree of freedom. This need not be in contradiction with the meaning we have attributed to it in the previous sections. We recall that in ordinary gauge theory a singular gauge transformation may convey some physical information. Now, looking at (2.22), the values $b = 0$ and $b = \infty$ may well correspond to singular gauge transformations, and therefore contain physical information. More generally the gauge nature of b may mean that using a different formulation one may be able to write the solution in terms of a single physical parameter which contains the information carried by both b and y .

The third question we would like to address is the relation between the two representations (5.9) and (5.19). The latter is expressed in terms of the open string center of mass x_0 and its interpretation is obvious. The interpretation of the former is less clear since the ‘time’ x does not have a clear connection with the open string center of mass time. A rather bold speculation is that x be connected with the closed string time. The closed string time couples to the closed string metric, which, in correspondence with the D-brane, must develop a singularity (it must be a solution of the effective low energy field theory associated to the closed string). So the relation between the open and the closed string time should be something like $g_c(dt_c)^2 \sim g_o(dt_o)^2$ in the field theory limit, were $g_o = 1$ and g_c becomes larger and larger near the origin. Something similar indeed occurs between x_0 and x when $\eta \rightarrow \infty$. In fact the ratio between x_0 and the zero mode part of x decreases exponentially with η . We notice moreover that the normalization of the representation (5.9) does not need any regularization. In other words x seems to be a smoother choice of time, with respect x_0 .

Next we would like to recall that recently, [19], the role of the time coordinate represented by the midpoint $X^0(\frac{\pi}{2})$ for causality in SFT has been emphasized. In our VSFT solution the profile along this time turns out to be highly singular: it is a constant infinite function (finite only at $X^0(\frac{\pi}{2}) = 0$), the inverse Wick–rotation of the midpoint space profile of [11].

To conclude, in this paper we have shown that VSFT contains solutions that describe brane decay with several features of the rolling tachyon. Of course a lot has still to be done. Further work is necessary to show a closer connection between our solution and the rolling tachyon. Moreover we may ask whether one can find solutions corresponding to half-S-branes. More important, the problems of energy conservation as well as the nature of the matter the branes decay into should be clarified.

Acknowledgments

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Appendix

A A collection of well-known formulae

In this Appendix we collect some useful results and formulas involving the matrices of the three strings vertex coefficients.

To start with, we recall that

- (i) V_{nm}^{rs} are symmetric under simultaneous exchange of the two couples of indices;
- (ii) they are endowed with the property of cyclicity in the r, s indices, i.e. $V^{rs} = V^{r+1,s+1}$, where $r, s = 4$ is identified with $r, s = 1$.

Next, using the twist matrix C ($C_{mn} = (-1)^m \delta_{mn}$), we define

$$X^{rs} \equiv CV^{rs}, \quad r, s = 1, 2, \quad (\text{A.1})$$

These matrices are often rewritten in the following way $X^{11} = X$, $X^{12} = X_+$, $X^{21} = X_-$. They commute with one another

$$[X^{rs}, X^{r's'}] = 0, \quad (\text{A.2})$$

moreover

$$CV^{rs} = V^{sr}C, \quad CX^{rs} = X^{sr}C \quad (\text{A.3})$$

Next we quote some useful identities:

$$\begin{aligned} X + X_+ + X_- &= 1 \\ X_+ X_- &= (X)^2 - X \\ (X_+)^2 + (X_-)^2 &= 1 - (X)^2 \\ (X_+)^3 + (X_-)^3 &= 2(X)^3 - 3(X)^2 + 1 \end{aligned} \quad (\text{A.4})$$

The same relations hold if we replace X, X_+, X_-, T by X', X'_+, X'_-, T' , respectively.

B Diagonal representation of CU'

With reference to formula (3.5), we illustrate the spectroscopy and diagonal representation of CU' . The matrix CU' is hermitian, unitary and commutes with $U'C$. The discrete eigenvalues ξ and $\bar{\xi}$ are determined as follows, [25]. Let

$$\xi = -\frac{2 - \cosh \eta - i\sqrt{3} \sinh \eta}{1 - 2\cosh \eta} \quad (\text{B.5})$$

and

$$F(\eta) = \psi\left(\frac{1}{2} + \frac{\eta}{2\pi i}\right) - \psi\left(\frac{1}{2}\right), \quad \psi(z) = \frac{d\log\Gamma(z)}{dz} \quad (\text{B.6})$$

Then the eigenvalues ξ and $\bar{\xi}$ are the solutions of

$$\Re F(\eta) = \frac{b}{4} \quad (\text{B.7})$$

The eigenvectors $V_n^{(\xi)}$ are defined via the generating function

$$\begin{aligned} F^{(\xi)}(z) = & \sum_{n=1}^{\infty} V_n^{(\xi)} \frac{z^n}{\sqrt{n}} = -\sqrt{\frac{2}{b}} V_0^{(\xi)} \left[\frac{b}{4} + \frac{\pi}{2\sqrt{3}} \frac{\xi-1}{\xi+1} + \log iz \right. \\ & \left. + e^{-2i(1+\frac{\eta}{\pi i})\arctan z} \Phi(e^{-4i\arctan z}, 1, \frac{1}{2} + \frac{\eta}{2\pi i}) \right] \end{aligned} \quad (\text{B.8})$$

where $\Phi(x, 1, y) = 1/y {}_2F_1(1, y; y+1; x)$, while

$$V_0^{(\xi)} = \left(\sinh \eta \frac{\partial}{\partial \eta} [\log \Re F(\eta)] \right)^{-\frac{1}{2}} \quad (\text{B.9})$$

As for the continuous spectrum, it is spanned by the variable k , $-\infty < k < \infty$. The eigenvalues of CU' are given by

$$\nu(k) = -\frac{2 + \cosh \frac{\pi k}{2} + i\sqrt{3} \sinh \frac{\pi k}{2}}{1 + 2 \cosh \frac{\pi k}{2}}$$

The generating function for the eigenvectors is

$$\begin{aligned} F_c^{(k)}(z) = & \sum_{n=1}^{\infty} V_n^{(k)} \frac{z^n}{\sqrt{n}} = V_0^{(k)} \sqrt{\frac{2}{b}} \left[-\frac{b}{4} - \left(\Re F_c(k) - \frac{b}{4} \right) e^{-k \arctan z} - \log iz \right. \\ & \left. - \left(\frac{\pi}{2\sqrt{3}} \frac{\nu(k)-1}{\nu(k)+1} + \frac{2i}{k} \right) + 2i f^{(k)}(z) - \Phi(e^{-4i\arctan z}, 1, 1 + \frac{k}{4i}) e^{-4i\arctan z} e^{-k \arctan z} \right] \end{aligned} \quad (\text{B.10})$$

where

$$F_c(k) = \psi(1 + \frac{k}{4\pi i}) - \psi(\frac{1}{2}),$$

while

$$V_0^{(k)} = \sqrt{\frac{b}{2\mathcal{N}(k)}} \left[4 + k^2 \left(\Re F_c(k) - \frac{b}{4} \right)^2 \right]^{-\frac{1}{2}} \quad (\text{B.11})$$

The continuous eigenvalues of X' , X'_+ , X'_- and T' (for the conventional lump) are given by same formulas as for the X , X_+ , X_- and T case, eqs(3.1,3.2). As for the discrete eigenvalues, they are given by the formulas

$$\begin{aligned} \mu_{\xi}^{rs} &= \frac{1 - 2\delta_{r,s} - e^{\eta} \delta_{r+1,s} - e^{-\eta} \delta_{r,s+1}}{1 - 2 \cosh \eta} \\ t_{\xi} &= e^{-|\eta|} \end{aligned} \quad (\text{B.12})$$

C Limits of X' and T'

In this Appendix we briefly discuss the low energy and high energy limit of X' and T' in the oscillator basis. The Neumann coefficients $V_{NM}^{(rs)}$ we use are given in Appendix B of [10]. They explicitly depend on the b parameter. In the low energy limit the three–strings vertex can be expanded by means of a parameter ϵ (a dimensionless parameter, in fact an alias of α'), see [11]. This translates into an expansion for $V_{NM}^{(rs)}$ triggered by the following rescalings

$$\begin{aligned} V_{mn}^{(rs)} &\rightarrow V_{mn}^{(rs)} \\ V_{m0}^{(rs)} &\rightarrow \sqrt{\epsilon} V_{m0}^{(rs)} \\ V_{00} &\rightarrow \epsilon V_{00} \end{aligned} \quad (C.13)$$

For instance X' is expanded as follows to the lowest orders of approximation

$$X' = \begin{pmatrix} -\frac{1}{3} + \frac{8}{3} V_{00} \frac{\epsilon}{b} & -\frac{4}{3} \sqrt{\frac{2\epsilon}{b}} \langle \mathbf{v}_e | \\ -\frac{4}{3} \sqrt{\frac{2\epsilon}{b}} |\mathbf{v}_e \rangle & X - \frac{8}{3} \frac{\epsilon}{b} (|\mathbf{v}_e \rangle \langle \mathbf{v}_e| - |\mathbf{v}_o \rangle \langle \mathbf{v}_o|) \end{pmatrix} \quad (C.14)$$

where

$$|\mathbf{v}_e \rangle_n = -\frac{3}{2\sqrt{2}} V_{0n}^{(11)}, \quad |\mathbf{v}_o \rangle_n = \sqrt{\frac{3}{8}} (V_{0n}^{(12)} - V_{0n}^{(21)})$$

It is interesting to remark that the parameter ϵ appears always divided by b , so that one could just as well absorb ϵ into $1/b$ and say that the expansion is in the parameter $1/b$ for large b . However to avoid confusion it is useful to keep the two parameters distinct.

Now, it is immediate to see that

$$T' = \begin{pmatrix} -1 + \mathcal{O}(\frac{\epsilon}{b}) & \mathcal{O}(\sqrt{\frac{\epsilon}{b}}) \\ \mathcal{O}(\sqrt{\frac{\epsilon}{b}}) & T + \mathcal{O}(\frac{\epsilon}{b}) \end{pmatrix} \quad (C.15)$$

This is correct provided we can prove that the use of (C.14) to compute T' makes full sense, that is all the terms of the expansion in powers of $\sqrt{\frac{\epsilon}{b}}$ are well defined. One can actually see that a naive expansion leads to infinite coefficients. This is a well-known problem, pointed out for the first time in [11], which requires a regularization. A nice way to introduce a regulator is to switch on a constant background B field. We will not do it here, but we quote the result: in the presence of a B field the infinities disappear, and the expansion (C.15) makes full sense. From this we deduce in particular that

$$T'_{nm} = T_{nm} + \mathcal{O}(\frac{\epsilon}{b}) \quad (C.16)$$

This result is used in Section 6.

Let us consider now another extreme expansion, that is the limit $\alpha' \rightarrow \infty$. In just the same way as above, we can introduce an alias, t ($t \gg 1$) instead of ϵ . So, in particular,

$$\begin{aligned} V_{mn}^{(rs)} &\rightarrow V_{mn}^{(rs)} \\ V_{m0}^{(rs)} &\rightarrow \sqrt{t} V_{m0}^{(rs)} \\ V_{00} &\rightarrow t V_{00} \end{aligned} \quad (C.17)$$

In this case X' to the lowest orders of approximation becomes

$$X' = \begin{pmatrix} 1 + \frac{2}{3} \frac{1}{V_{00}} \frac{b}{t} & -\frac{2}{3} \sqrt{\frac{2b}{t}} \langle \mathbf{v}_e \rangle \\ -\frac{2}{3} \sqrt{\frac{2b}{t}} \langle \mathbf{v}_e \rangle & X - \frac{4}{3} \frac{1}{V_{00}} \left(1 - \frac{1}{V_{00}} \frac{b}{2t}\right) (\langle \mathbf{v}_e \rangle \langle \mathbf{v}_e \rangle - \langle \mathbf{v}_o \rangle \langle \mathbf{v}_o \rangle) \end{pmatrix} \quad (C.18)$$

The lowest order in this expansion is known as the tensionless limit [32]. Also here one must be careful about the use of this expansion in calculating T' . From eq.(C.18) one finds that

$$T'_{00} = 1 + \mathcal{O}\left(\frac{b}{t}\right) \quad (C.19)$$

D The $\alpha' \rightarrow 0$ limit of $\check{S}_{nm}^{(c)}$ and $\check{S}_{0n}^{(c)}$

In this Appendix we discuss the limit of the unconventional lump matrix elements $\check{S}_{nm}^{(c)}$ and $\check{S}_{0n}^{(c)}$ by means of the diagonal basis. According to (3.12), we speak interchangeably of the $b \rightarrow \infty$ limit and the $\eta \rightarrow \infty$ one. When applying the results of this Appendix to section 6, we understand that $1/b$ is replaced everywhere by ϵ/b with finite b .

As a preliminary step let us prove that

$$\lim_{b \rightarrow \infty} \left(V_0^{(k)} \right)^2 = \delta(k) \quad (D.20)$$

A rather informal way to see this is as follows. Looking at (B.11) it is easy to realize that the limit always vanishes provided $k \neq 0$. Therefore the support of the limiting distribution must be at $k = 0$. We can therefore expand all the functions involved in k around $k = 0$ and keep the leading terms. Since $\Re F_c(k) \approx 1.386\dots$ around this point, we can disregard $\Re F_c(k)$ compared to $b/4$ in the $b \rightarrow \infty$ limit. Therefore we easily find

$$\lim_{b \rightarrow \infty} (V_0^{(k)})^2 = \lim_{b \rightarrow \infty} = \frac{\bar{b}}{\pi} \frac{1}{1 + \bar{b}^2 k^2}$$

where $\bar{b} = b/8$. Now defining $\bar{\epsilon} = 1/\bar{b}$, the limit becomes

$$\lim_{\bar{\epsilon} \rightarrow 0} \frac{1}{\pi} \frac{\bar{\epsilon}}{k^2 + \bar{\epsilon}^2} = \delta(k) \quad (D.21)$$

according to a well-known representation of the delta function. We can also show that

$$(V_0^{(k)})^2 = \delta(k) + \mathcal{O}(1/b)$$

From now on we suppose that, in the $\int dk$ integrals, we are allowed to replace the integrands with their $1/b$ expansions, and that the results we obtain are valid at least in an asymptotic sense. This attitude is always confirmed by numerical approximations.

D.1 Limit of $\check{S}_{mn}^{'(c)}$

Let us rewrite the generating function for $V_m^{(k)}$ as follows:

$$F^{(k)}(z) = A^{(k)} f^{(k)}(z) - \frac{(1 - \nu(k))V_0^{(k)}}{\sqrt{b}} B(k, z) \quad (\text{D.22})$$

where

$$A^{(k)} = V_0^{(k)} \sqrt{\frac{2}{b}} k \left(\Re F_c(k) - \frac{b}{4} \right) \quad (\text{D.23})$$

and

$$\begin{aligned} B(k, z) &= \frac{2}{1 - \nu(k)} \left[\Re F_c(k) + \frac{\pi}{2\sqrt{3}} \frac{\nu(k) - 1}{\nu(k) + 1} + \frac{2i}{k} + \log(iz) - 2if^{(k)}(z) \right. \\ &\quad \left. + \text{LerchPhi}(e^{-4i\arctan(z)}, 1, 1 + \frac{k}{4i}) e^{-4i\arctan(z)} e^{-karctan(z)} \right] \end{aligned} \quad (\text{D.24})$$

From (D.22) we can derive a useful expression for $V_m^{(k)}$:

$$V_m^{(k)} = A^{(k)} \frac{\sqrt{m}}{2\pi i} \oint dz \frac{f^{(k)}(z)}{z^{m+1}} - \frac{(1 - \nu(k))V_0^{(k)}}{\sqrt{b}} \frac{\sqrt{m}}{2\pi i} \oint dz \frac{B(k, z)}{z^{m+1}} \quad (\text{D.25})$$

Since $v_m^{(k)} = \frac{\sqrt{m}}{2\pi i} \oint dz \frac{f^{(k)}(z)}{z^{m+1}}$ and $S_{mn}^{'(c)} = \int_{-\infty}^{\infty} dk t(k) V_m^{(k)} V_n^{(-k)}$ we get:

$$\begin{aligned} \check{S}_{mn}^{'(c)} &= \int_{-\infty}^{\infty} dk t(k) \left[A^{(k)} A^{(-k)} v_m^{(k)} v_n^{(-k)} - A^{(k)} V_0^{(k)} v_m^{(k)} (1 - \bar{\nu}(k)) \tilde{B}_n(-k) \frac{1}{\sqrt{b}} \right. \\ &\quad \left. - A^{(-k)} V_0^{(k)} v_n^{(-k)} (1 - \nu(k)) \tilde{B}_m(k) \frac{1}{\sqrt{b}} + (V_0^{(k)})^2 (1 - \bar{\nu}(k)) (1 - \nu(k)) \tilde{B}_m(k) \tilde{B}_n(-k) \frac{1}{b} \right] \end{aligned} \quad (\text{D.26})$$

where

$$\tilde{B}_m(k) = \frac{\sqrt{m}}{2\pi i} \oint dz \frac{B(k, z)}{z^{m+1}}$$

Now we want to take the limit of (D.26) when $b \rightarrow \infty$. To this end we notice the following:

$$\begin{aligned}\lim_{b \rightarrow \infty} A^{(k)} A^{(-k)} &= \lim_{b \rightarrow \infty} (V_0^{(k)})^2 \left(\frac{-2k^2}{b} \right) \left(\Re F_c(k) - \frac{b}{4} \right)^2 \\ &= \lim_{x \rightarrow -\infty} \left(\frac{-k^2}{N(k)} \right) \frac{x^2}{4 + k^2 x^2} = \left(\frac{-k^2}{N(k)} \right) \frac{1}{k^2} = -\frac{1}{N(k)}\end{aligned}$$

where $x = (\Re F_c(k) - \frac{b}{4})$. When k is very large $\Re F_c(k)$ tends to (slowly) diverge, but the factor $t(k)$ in the integrand of (D.26) concentrates the integral in the small k region.

We also need:

$$\begin{aligned}\lim_{b \rightarrow \infty} \frac{A^{(k)} V_0^{(k)}}{\sqrt{b}} &= \sqrt{2} k \delta(k) \left(\frac{\Re F_c(k)}{b} - \frac{1}{4} \right) \\ \lim_{b \rightarrow \infty} \frac{A^{(-k)} V_0^{(k)}}{\sqrt{b}} &= -\sqrt{2} k \delta(k) \left(\frac{\Re F_c(k)}{b} - \frac{1}{4} \right)\end{aligned}$$

Finally using these limits

$$\begin{aligned}\lim_{b \rightarrow \infty} \check{S}_{mn}'^{(c)} &= - \int_{-\infty}^{\infty} \frac{dk}{N(k)} t(k) v_m^{(k)} v_n^{(-k)} \\ &\quad + \lim_{b \rightarrow \infty} \int_{-\infty}^{\infty} dk t(k) \delta(k) (1 - \bar{\nu}(k)) (1 - \nu(k)) \tilde{B}_m(k) \tilde{B}_n(-k) \frac{1}{b}\end{aligned}$$

while the other integrals vanish because they contain the factor $k \delta(k)$. Here we have used the fact that $\nu(0) = \bar{\nu}(0) = -1$ and $\tilde{B}_m(0)$ is finite, for a straightforward calculation gives

$$\tilde{B}_m(0) = \frac{\sqrt{m}}{2\pi i} \oint dz \frac{\log(1+z^2)}{z^{m+1}} = \begin{cases} 0 & \text{for } m \text{ odd;} \\ \frac{\sqrt{2m}}{2} (-1)^{\frac{m}{2}+1} \left(\frac{m}{2}+1\right)! & \text{for } m \text{ even.} \end{cases} \quad (\text{D.27})$$

So we are left with:

$$\lim_{b \rightarrow \infty} \check{S}_{mn}'^{(c)} = S_{mn} \quad (\text{D.28})$$

This is the sliver. The corrections are of order $\frac{1}{b}$.

D.2 Limit of \check{S}_{0m}'

In the rest of this appendix we would like to justify eq.(6.2). The limit of $\check{S}_{0m}'^{(c)}$ can be computed the same way as before. We have:

$$\lim_{b \rightarrow \infty} \check{S}_{0m}'^{(c)} = \lim_{b \rightarrow \infty} \int_{-\infty}^{\infty} dk t(k) V_0^{(k)} V_m^{(-k)} = \lim_{b \rightarrow \infty} \left(\int_{-\infty}^{\infty} dk t(k) V_0^{(k)} A^{(-k)} v_m^{(-k)} + \frac{2}{\sqrt{b}} \tilde{B}_m(0) \right) \quad (\text{D.29})$$

The last term in the RHS of course vanishes in the limit $b \rightarrow \infty$, while the first limit diverges, but, recalling (6.2), what we are really need to know is the limit of $\frac{\check{S}'_{0m}}{1+s'}$. Using the fact that $1+s' \approx 4\eta \log \eta$ when $b \rightarrow \infty$ ($b \approx 4\log \eta$) and that we can write $\check{S}'_{0m} = \check{S}'_{0m}^{(c)} + \check{S}'_{0m}^{(d)}$ (factorization into continuous and discrete parts) we have:

$$\begin{aligned}\check{S}'_{0m}^{(d)} &\approx 2\eta\sqrt{2\log \eta} \\ \check{S}'_{0m}^{(c)} &\approx \int_{-\infty}^{\infty} dk t(k) v_m^{(-k)} (-\sqrt{2}k) (V_0^{(k)})^2 \left(\frac{\Re F_c(k)}{4\log \eta} - \frac{1}{4} \right) 2\sqrt{\log \eta}\end{aligned}$$

Using these we get:

$$\frac{\check{S}'_{0m}^{(c)}}{1+s'} \approx \int_{-\infty}^{\infty} dk t(k) v_m^{(-k)} (\sqrt{2}k) \delta(k) \left(\frac{\Re F_c(k)}{4\log \eta} - \frac{1}{4} \right) \frac{1}{2\eta} = 0$$

and

$$\frac{\check{S}'_{0m}^{(d)}}{1+s'} \approx \frac{1}{\sqrt{2\log \eta}}$$

Hereby the conclusion (6.2) follows.

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